

Thick-wall Cylinders:

Let:

r_i = inside radius of a cylinder

r_o = outside radius

p_i = internal pressure

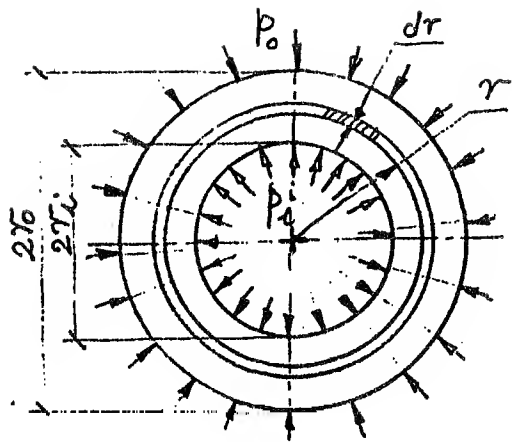
p_o = external pressure

σ_t = tangential stress

σ_r = radial stress

σ_l = longitudinal stress

Principle stresses



Then:

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i) / r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i) / r^2}{r_o^2 - r_i^2}$$

+ve values indicate tension

-ve values indicate compression

Also:

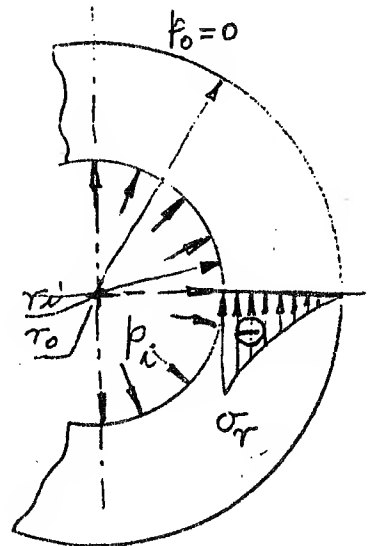
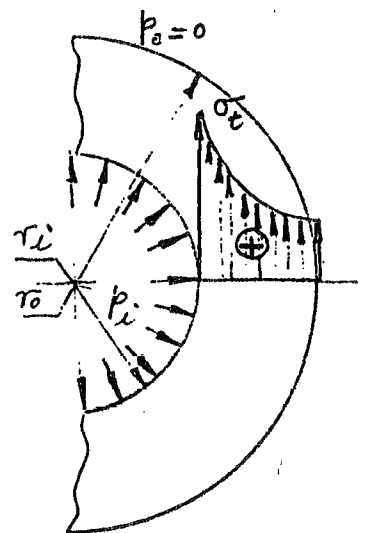
For $p_o = 0$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2} \right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r^2} \right)$$

When the cylinder has closed ends:

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$



Interference Fits:

Let:

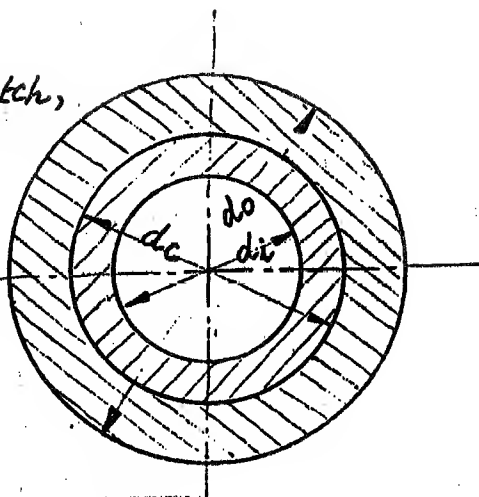
d_i, d_o, d_c are as shown in sketch,

p = pressure at contact surface.

δ = the total interference.

μ_i, μ_o = Poisson's ratio for inner and outer cylinders material respectively.

E_i, E_o = Modulus of elasticity for inner and outer cylinders material respectively.



Then:

$$p_c = \frac{\delta}{d_c \left[\frac{d_c^2 + d_i^2}{E_i(d_c^2 - d_i^2)} + \frac{d_o^2 + d_c^2}{E_o(d_o^2 - d_c^2)} - \frac{\mu_i}{E_i} + \frac{\mu_o}{E_o} \right]} \quad \text{--- (1)}$$

Cases:

(1) $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2d_c^3(d_o^2 - d_i^2)}{(d_c^2 - d_i^2)(d_o^2 - d_c^2)} \right]}$$

(2) $d_i = 0$, i.e. solid inner cyl., $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2d_c d_o^2}{d_o^2 - d_c^2} \right]}$$

{ e.g. inner race of a bearing on a solid shaft, a coupling hub or gear on a solid shaft, ..

(3) $d_o = \infty$, $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2d_c^3}{(d_c^2 - d_i^2)} \right]}$$

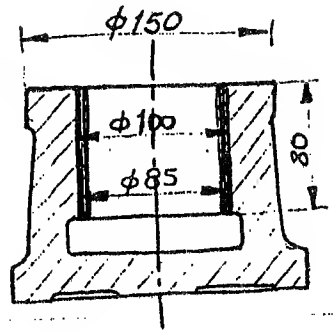
{ e.g. outer race of a bearing in a housing, a bearing bush or a gear nut of a power screw, ..

Note: if the materials are different, which is the general case, use equation (1) by substituting $d_i = 0$ for case (2) and dividing by d_o^2 and getting the limit value in case (3)

Force fitting:

Calculate the force required to drive a Tin-Bronze bush into the CI-foot pedestal of a jib crane if:
 $\mu_i = 0.3, E_i = 93 \times 10^3 \text{ N/mm}^2$ for G-Cu Sn 12 and
 $\mu_o = 0.27, E_o = 80 \times 10^3 \text{ N/mm}^2$ for C.I. GG.22

The type of fit is H_8/n_7 OR H_7/n_6
 $\mu \text{ st/CI} = \mu \text{ Br/CI} = 0.15 - 0.25$ friction coeff.



$\phi 100 H_8/n_7$

$$\begin{array}{l} \text{Hole } 100 \begin{array}{l} +0.054 \\ -0.000 \end{array}, \text{ bush O.D. } 100 \begin{array}{l} +0.058 \\ +0.023 \end{array} \\ \text{average } +0.027 \qquad \qquad \text{average } +0.041 \\ \delta = 14 \mu \end{array}$$

$\phi 100 H_7/n_6$

$$\begin{array}{l} \text{Hole } 100 \begin{array}{l} +0.035 \\ -0.000 \end{array}, \text{ bush O.D. } 100 \begin{array}{l} +0.045 \\ +0.023 \end{array} \\ \text{average } +0.018 \qquad \qquad \text{average } +0.034 \\ \delta_2 = 16 \mu \end{array}$$

$$d_i = 85 \text{ mm}, d_c = 100 \text{ mm}, d_o = 150 \text{ mm}, l = 80 \text{ mm}$$

a) Contact pressure

$$p_c = \frac{\delta}{d_c \left[\frac{(d_c^2 + d_i^2)}{E_i(d_c^2 - d_i^2)} + \frac{d_o^2 + d_c^2}{E_o(d_o^2 - d_c^2)} - \frac{\mu_i}{E_i} + \frac{\mu_o}{E_o} \right]}$$

$$\frac{d_c^2 + d_i^2}{E_i(d_c^2 - d_i^2)} = \frac{(100)^2 + (85)^2}{93 \times 10^3 (100^2 - 85^2)} = 0.6674 \times 10^{-4}$$

$$\frac{d_o^2 + d_c^2}{E_o(d_o^2 - d_c^2)} = \frac{150^2 + 100^2}{80 \times 10^3 (150^2 - 100^2)} = 0.325 \times 10^{-4}$$

$$\frac{\mu_i}{E_i} = \frac{0.3}{93 \times 10^3} = 0.0323 \times 10^{-4}, \frac{\mu_o}{E_o} = \frac{0.27}{80 \times 10^3} = 0.034 \times 10^{-4}$$

$$p_{c_1} = \frac{0.014}{100} \left[\frac{10^4}{0.6674 + 0.325 - 0.0323 + 0.034} \right] = 1.408 \text{ N/mm}^2$$

also

$$p_{c_2} = 1.6095 \text{ N/mm}^2$$

b) Required driving force :

$$A_c = \text{Contact area} = \pi(100)(80) = 8000\pi \text{ mm}^2$$

$$\text{Driving force} = F = p_c * A_c * \mu$$

$$\therefore F_1 = 1.408 * (8000\pi) * 0.25 * 10^{-3} = \boxed{8.85} \text{ kN}$$

$$F_2 = 1.6095 * (8000\pi) * 0.25 * 10^{-3} = \boxed{10.11} \text{ kN}$$

Note: Two other fits are investigated

	H_7/p_6		H_7/s_6
Hole	$100 \begin{matrix} +0.035 \\ -0.000 \end{matrix} \Big] +0.018$		$100 \begin{matrix} +0.035 \\ -0.000 \end{matrix} \Big] +0.018$
Bush	$100 \begin{matrix} +0.059 \\ +0.037 \end{matrix} \Big] +0.048$		$100 \begin{matrix} +0.093 \\ +0.071 \end{matrix} \Big] +0.082$

average δ 0.030 mm 0.082 mm

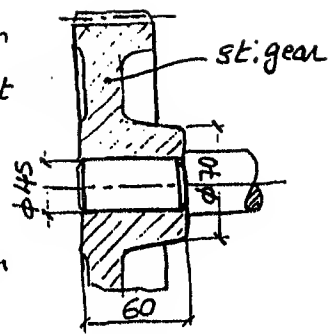
Contact press. 3.017 N/mm² 8.247 N/mm²

Driving force $\boxed{\approx 19}$ kN $\boxed{51.8}$ kN

The framed values of the required driving force reflect the drastic change of the force as affected by the amount of the interference employed.

Interference fit to transmit torque :

15 kW at 1450 rpm is to be transmitted from a gear-wheel hub to a 45 mm diam solid shaft as shown by the sketch. The security factor is considered 2.0 (starting under full load), the friction coefficient = 0.15 (st/st minimum dry friction coefficient).



Calculate the required interference and the type/grade of fit to allow this based on the average interference. (as the probable expected interference).

$$\text{Torque} = \frac{71620 * \text{H.P.}}{n} = \frac{71620 * 15 * 1.36 * 9.81 * 10}{1450} = 99 \text{ kN.m}$$

$$\text{Design torque} = 2.0 * 99 \cong 200 \text{ kN.m}$$

$$T_{\text{design}} = p_c (\pi d_c) l_c * \frac{d_c}{2} * \mu$$

$$\therefore 200,000 = p_c (\pi * 45) (60) \left(\frac{45}{2}\right) (0.15), p_c = \text{contact pressure}$$

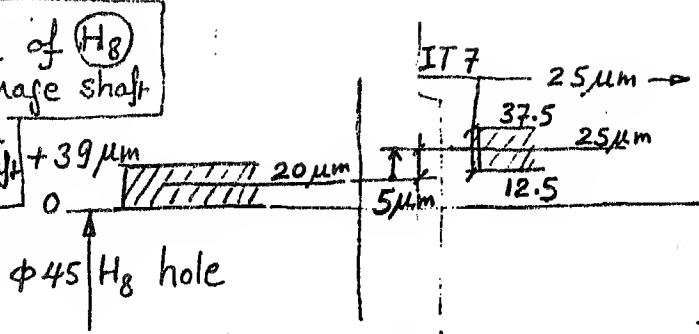
$$\therefore p_c \cong 7.0 \text{ N/mm}^2$$

$$p_c \left[\frac{2 d_c d_o^2}{d_o^2 - d_c^2} \right] = \delta E$$

$$(7.0) \left[\frac{2 * 45 * 70^2}{70^2 - 45^2} \right] = \delta (2.1 * 10^5) \therefore \delta = 5.11 * 10^{-3} \text{ mm}$$

$$\therefore \delta = 5 \text{ } \mu\text{m}$$

- get average hole of H_8
- add δ to get average shaft
- Apply IT_7 on both sides of average shaft
- Look for type of shaft of grade 7 that have upper & lower limit



The required shaft tolerance based on IT_7 is $+37.5$ with an average of $+25 \mu$ for a diameter = 45 mm.

$$45 \text{ (m}_7) = 45 \begin{matrix} +34 \mu \\ +9 \mu \end{matrix} \text{] - average } \frac{43}{2} = 21.5 \mu$$

$$45 \text{ (n}_7) = 45 \begin{matrix} +42 \mu \\ +17 \mu \end{matrix} \text{] - average } \frac{59}{2} = 29.5 \mu$$

$$45 \text{ (p}_7) = 45 \begin{matrix} +51 \mu \\ +26 \mu \end{matrix} \text{] - average } \frac{76}{2} = 36 \mu$$

$45 \text{ n}_7 = 45 \begin{matrix} +0.042 \\ +0.017 \end{matrix}$ is the type and grade nearest to the required average of 25μ that guarantee a min. average interference of $9.5 \mu > 5 \mu$ calculated.

In such a case:

$$p_{c1} = 7 * \frac{9.5}{5} = 13.3 \text{ N/mm}^2$$

$$\begin{aligned} \text{Force required for fitting} &= p_c * 0.25 * \pi * 45 * 60 \\ &= 28.2 \text{ kN} \end{aligned}$$

Note: Force fitting will hurt the contact surfaces. It advisable to utilize heating the gear hub (or cooling the shaft)

$$\Delta l = l \alpha \Delta t, \quad \Delta l = \text{average interference} = 9.5 * 10^{-3} \text{ mm}$$

$$l = \text{diam} = 45 \text{ mm}$$

$$\alpha = 11 * 10^{-6} \text{ mm/mm}^\circ\text{C}$$

$$\therefore \Delta t = \frac{9.5 * 10^{-3}}{45 * 11 * 10^{-6}} = 19^\circ\text{C}$$

ambient = 35°C , losses during fitting duration $\approx 15^\circ\text{C}$

$$\therefore \text{heat to } t = 19 + 35 + 15 = 69^\circ\text{C} \approx \underline{70^\circ\text{C}}$$